Book of Abstracts

Dipartimento
di Matematica
e Informatica

Workshop

FOURIER ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS II



Ferrara, 29th - 30th January 2024

Speakers:

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Vicente Asensio López
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Fernando de Ávila Silva
Carmen Fernández
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Supported by GNAMPA-INdAM and Università di Ferrara

Multipliers and Convolutors of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ of the ω-ultradifferentiable rapidly decreasing functions of Beurling type and the action of the Fourier transform

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In the last years the attention has focused on the space $\mathcal{S}_\omega(\mathbb{R}^N)$ of the ultradifferentiable rapidly decreasing functions of Beurling type, as defined by Björck. Following this line of research, we introduce and study the space $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ ($\mathcal{O}_{C,\omega}(\mathbb{R}^N)$) of the slowly increasing functions (of the very slowly increasing functions) of Beurling type in the setting of ultradifferentiable function space in the sense of Braun, Meise and Taylor. We show that $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$ is the space of the multipliers of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ and of its dual space $\mathcal{S}'_\omega(\mathbb{R}^N)$. We also show that the strong dual $\mathcal{O}'_{C,\omega}(\mathbb{R}^N)$ of $\mathcal{O}_{C,\omega}(\mathbb{R}^N)$ is the space of convolutors of the space $\mathcal{S}_\omega(\mathbb{R}^N)$ and of its dual space $\mathcal{S}'_\omega(\mathbb{R}^N)$. Moreover, we establish that the Fourier transform is an isomorphism from $\mathcal{O}'_{C,\omega}(\mathbb{R}^N)$ onto $\mathcal{O}_{M,\omega}(\mathbb{R}^N)$.

Some remarks on the Cauchy Problem for Schrödinger type equations in Gelfand-Shilov spaces

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We consider the Cauchy problem

$$\begin{cases} S(t,x,D_t,D_x)u(t,x) = f(t,x),\ (t,x) \in [0,T] \times \mathbb{R}, \\ u(0,x) = g(x),\ x \in \mathbb{R}, \end{cases} \tag{1}$$

where

$$S(t,x,D_t,D_x) = D_t - \Delta + \sum_{j=1}^{n} a_j(t,x) D_{x_j} + b(t,x),$$

and $a_j(t,x), b(t,x)$ are continuous with respect to time and Gevrey regular with respect to space variable x.

In this talk, we shall discuss a sufficient decay condition at $|x| \to \infty$ on the coefficients $a_j(t,x)$ to obtain well-posedness for the Cauchy problem (1) in Gelfand-Shilov spaces.

The Wigner global wave front set in ultradifferentiable classes

Vicente Asensio

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In this talk we introduce global wave front sets using time-frequency analysis tools defined in global classes of ultradifferentiable functions of Beurling type modulated with weight functions in the sense of Braun, Meise, and Taylor. We focus on the wave front set given by the Wigner transform, relating it to others in the literature. We also show that these wave front sets can be described by the decayment of some coefficients in sufficiently dense lattices. Finally, we compute the wave front set for some concrete distributions.

An application of the Wigner transform to Quasicrystals of Fourier type

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Quasicrystals were discovered by Daniel Schechtman in 1982 and are a form of ordered matter which was previously thought to be impossible. Several mathematical models for their study have since been proposed. One of them are *Fourier quasicrystals*, defined as tempered distributions μ satisfying symmetric conditions on μ and $\widehat{\mu}$. This symmetry suggests a profitable use of time-frequency analysis and in this direction we present results obtained by posing "quasicrystals type" conditions on time-frequency representations, instead of separately on the distribution and its Fourier transform. More precisely we show that a tempered distribution μ on $\mathbb{R}^d$ whose Wigner transform, $W(\mu)$, is supported on a product of two uniformly discrete sets in $\mathbb{R}^d$ is a Fourier quasicrystal. Furthermore we present some extensions of this result to matrix-Wigner transforms, which include most of the commonly used time-frequency representations.

Global hypoellipticity and solvability for evolution operators in Gelfand-Shilov spaces

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We discuss necessary and sufficient conditions for global hypoellipticity and for global solvability in Gelfand-Shilov spaces of a class of evolution operators with coefficients depending on time and space variables (t, x) and which are periodic with respect to t and polynomially bounded with respect to x.

Wigner Representation of Schrödinger Propagators

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We perform a Wigner analysis of Fourier integral operators (FIOs), whose main examples are Schrödinger propagators arising from quadratic Hamiltonians with bounded perturbations. The perturbation is given by a pseudodifferential operator $\sigma(x, D)$ with symbol in the Hörmander class $S^0_{0,0}(\mathbb{R}^{2d})$. We compute and study the Wigner kernel of these operators, that fall in a more general abstract class of FIOs named $FIO(S)$, with $S \in Sp(d, \mathbb{R})$ the symplectic matrix representing the classical symplectic map. We shall show the algebra and the Wiener's property of this class. The algebra will be the fundamental tool to represent the Wigner kernel of the Schrödinger propagator for every $t \in \mathbb{R}$, also in the caustic points. This outcome underlines the validity of the Wigner analysis for the study of Schrödinger equations.

Boundedness of pseudodifferential and Fourier integral operators on Orlicz spaces

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We study the action of pseudodifferential and Fourier integral operators on Orlicz spaces $L^\Phi(\mathbb{R}^d)$ and weighted spaces modelled on $L^\Phi(\mathbb{R}^d)$. We also prove propagation of global wave-front sets, with such spaces as reference spaces, under the action of operators defined by means of symbols satisfying global estimates on $\mathbb{R}^d$. In particular, we illustrate the role of the Lebesgue exponents p_Φ and q_Φ, associated with the Young function Φ, in proving such results.

Sharp $L^p - L^q$ estimates for an oscillatory-diffusive multiplier

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In this talk, we employ the Fourier analysis to derive sharp $L^p - L^q$ estimates, $1 \leqslant p \leqslant q \leqslant \infty$, for a parameter-dependent multiplier in the form

$$m(\tau, \xi) = e^{-(\tau|\xi|)^\theta} \operatorname{sinc}|\xi|,$$

where $\theta > 0$ and $\xi \in \mathbb{R}^n$. This estimate is the key step used to prove $L^p - L^q$ estimates for a class of dissipative wave equations,

$$u_{tt} - \Delta u + Au_t = 0, \quad t > 0, \ x \in \mathbb{R}^n,$$

in the so-called "overdamping" regime [1, Lemma 1].

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A class of globally hypoelliptic systems of periodic pseudodifferential operators

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We present an investigation of the global hypoellipticity problem for a class of systems of pseudodifferential operators on the torus. The approach consists of establishing conditions on the matrix symbol of the system such that it can be transformed into a suitable triangular form with constant coefficients. In particular, we show that an obstruction of number-theoretical nature appears as necessary and sufficient conditions for global hypoellipticity.

Composition operator on Gelfand-Shilov classes

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The aim of this talk is to present joint work with H. Ariza and A. Galbis concerning composition operators on global classes of ultradifferentiable functions of Beurling type invariant under Fourier transform, in particular on the classical Gelfand-Shilov classes $\Sigma_d(\mathbb{R})$, $d > 1$.

As it happens with the Schwartz class, composition operators in this setting are never compact. From this point, the behaviour of composition operators on Gelfand-Shilov spaces and on the Schwartz class is completely different: for example a necessary condition for the composition operator C_ψ to leave the class $\Sigma_d(\mathbb{R})$, $d > 1$, invariant is the boundedness of ψ'. We also find the optimal index d' for which $C_\psi(\Sigma_d(\mathbb{R})) \subset \Sigma_{d'}(\mathbb{R})$ holds for any non-constant polynomial ψ.

Pseudodifferential operators with completely periodic symbol and applications to Gabor frames

Gianluca Garello (joint work with Alessandro Morando)

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A Gabor system $\mathcal{G}(g,a,b)$ is a sequence of the type $\{g_{hk} = e^{2\pi i\, bk\cdot x} g(x-ah)\}_{h,k\in\mathbf{Z}^d}$, with g measurable function on $\mathbf{R}^d$, $a,b>0$. $\mathcal{G}(g,a,b)$ is said to be a frame in $L^2(\mathbf{R}^d)$ if $A\|f\|^2_{L^2} \leqslant \sum_{h,k\in\mathbf{Z}^d} |(f,g_{h,k})|^2 \leqslant B\|f\|^2_{L^2}$, for some $A,B>0$ and any $f\in L^2(\mathbf{R}^d)$. Gabor frames play an important role in signal processes.

A wide literature, see for example [2], [3], [4], [5], is devoted in finding conditions on the window g and the lattice parameters $a,b>0$, which allow the corresponding Gabor system to be a frame in $L^2(\mathbf{R}^d)$, so that the Gabor operator $S_{g,g}f = \sum_{h,k\in\mathbf{Z}^d}(f,g_{hk})g_{hk}$ is invertible in $\mathcal{L}(L^2)$ and a reconstruction formula $f = \sum_{h,k\in\mathbf{Z}^d}(f,g_{hk})\gamma_{h,h}$ is available, with $\gamma = S^{-1}_{g,g}g$. To this respect the very basic assumption is that a and b are "small enough".

In this talk we show sufficient conditions which allow $\mathcal{G}(g,a,b)$ to be a frame, by using the results of continuity and invertibility in $L^p(\mathbf{R}^d)$ for pseudodifferential operators with symbols $\sigma(x,\xi)$ periodic in both the variable, introduced in [1].

References

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Higher order hyperbolic equations with multiplicities

Claudia Garetto (joint work with Michael Ruzhanksy and Bolys Sabibtek)

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In this talk I will discuss Gevrey and C^∞ well-posedness for linear higher order hyperbolic equations with multiplicities. I will review the different methods employed for time- and/or x-dependent coefficients and the conditions needed on the lower order terms.

Compactness of the Weyl operator in $\mathcal{S}_\omega$

David Jornet (joint work with Vicente Asensio, Chiara Boiti and Alessandro Oliaro)

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We study the compactness of the Weyl operator in $\mathcal{S}_\omega$ in terms of its symbols and its kernel. Some examples are given.

Boundary values of zero solutions of hypoelliptic differential operators in ultradistribution spaces

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We discuss recent results on ultradistributional boundary values of zero solutions of a hypoelliptic constant coefficient partial differential operator. These results unify and considerably extend various classical results of Komatsu and Matsuzawa about boundary values of holomorphic functions, harmonic functions and zero solutions of the heat equation in ultradistribution spaces.

References

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Scattering for defocusing Semilinear Klein-Gordon equation with a magnetic field

Sandra Lucente (joint work with Luca Fanelli and Vladimir Georgiev)

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In this talk we will collect the most relevant results on Cauchy Problem and Scattering for the solution $u = u(t, x)$ of

$$(\partial_t^2 - \Delta + m^2)u = f(u)$$

according to the sign and the growth of $|f(u)| \simeq |u|^p$ and the space dimension.

Starting from this, we will describe some results obtained in a joint project with Luca Fanelli and Vladimir Georgiev. More precisely, we will consider the following variant of the equation:

$$(\partial_t^2 - (\nabla + iA) \cdot (\nabla + iA) + m^2)u = f(u)$$

with suitable magnetic fields $A = A(t, x)$. The results involves Morawetz type estimates and localized energy technique.

Generalized Gaussian Estimates for Elliptic Operators with Unbounded Coefficients on Domains

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Gaussian estimates for heat kernels $k(t,\cdot,\cdot)$ of semigroups in spaces $L^2(\Omega)$ associated to strictly elliptic operators with bounded coefficients are nowaday a classical and well investigated topic, mainly for their relevant consequences on the regularity of the semigroups, For example, they allow to prove the analyticity of the semigroup on the L^p-scale on a sector which is independent of p.

In this talk, after reviewing some classical results, we consider a smooth enough domain $\Omega \subseteq \mathbb{R}^d$ and non-self-adjoint operators in divergence form, defined on smooth functions u by

$$\mathcal{A}u = \mathrm{div}(Q\nabla u) - (B, \nabla u) + \mathrm{div}(Cu) - Vu,$$

where the real-valued measurable coefficients $Q = (q_{ij})$, $B = (B_i)_i$, $C = (C_i)_i$, $(i, j = 1, \dots, d)$ and V belong to $L^\infty_{\mathrm{loc}}(\Omega)$ and are allowed to be unbounded. A version of Gaussian estimates for the semigroup generated by the realization of $\mathcal{A}$ in $L^2(\Omega)$, subject to Neumann-type boundary conditions, is presented and consequences for the regularity of the semigroup are discussed.

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Mean-dispersion principles and the Wigner transform

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In this talk we present uncertainty principles for families of orthonormal functions in $L^2(\mathbb{R})$ in connection with time-frequency analysis. When talking about uncertainty principles, in harmonic analysis, one refers to a class of theorems giving limitations on how much a function and its Fourier transform can be both localized at the same time. Different meanings of the word "localized" give rise to different uncertainty principles. There are, moreover, uncertainty principles giving not only limitations on the localization of a single function and its Fourier transform, but on how such limitations behave, becoming stronger and stronger, when adding more and more elements of an orthonormal system in L^2. In this talk we focus in particular on results of this type involving means and variances. For $f \in L^2(\mathbb{R})$ we define the *associated mean* and the *associated variance* by

$$\mu(f) := \frac{1}{\|f\|^2}\int_{\mathbb{R}} t|f(t)|^2 dt \quad \text{and} \quad \Delta^2(f) := \frac{1}{\|f\|^2}\int_{\mathbb{R}} |t-\mu(f)|^2|f(t)|^2 dt,$$

respectively. The *dispersion* associated with f is $\Delta(f) := \sqrt{\Delta^2(f)}$. An uncertainty principle for orthonormal sequences, that constitutes the starting point of the present work, is due to Shapiro.

Theorem (Shapiro's Mean-Dispersion Principle). There does not exist an infinite orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ such that all $\mu(f_k)$, $\mu(\hat{f_k})$, $\Delta(f_k)$, $\Delta(\hat{f_k})$ are uniformly bounded.

Some refinements of this result have been obtained in the literature, and a quantitative version of it has been proved by Jaming and Powell in [1].

In this talk we present uncertainty principles of mean-dispersion type involving quadratic time-frequency representations (in particular the Wigner transform) applied to the elements of an orthonormal system in $L^2(\mathbb{R})$. Such results are given in a quantitative form, including in particular the results of [1] and the classical Shapiro's mean-dispersion principle. In particular, if $\{f_k\}_{k\in\mathbb{N}_0}$ is an orthonormal sequence in $L^2(\mathbb{R})$, we prove that for every $n \geqslant 0$

$$\sum_{k=0}^{n}\int_{\mathbb{R}^2}(x^2+\xi^2)|W(f_k)(x,\xi)|^2 dx d\xi \geqslant \frac{(n+1)^2}{2}.$$

As a consequence, we show that there does not exist an infinite orthonormal sequence $\{f_k\}_{k\in\mathbb{N}_0}$ in $L^2(\mathbb{R})$ satisfying $\mu(f_k) = \mu(\hat{f_k}) = 0$ such that all the traces of the covariance matrices of $|W(f_k)(x,\xi)|^2$ are uniformly bounded.

References

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On a nonlinear wave equation in the contracting de Sitter spacetime with a critical nonlinearity

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In this talk, we discuss a semilinear wave equation in the contracting de Sitter spacetime. We focus on blow-up results and upper-bound estimates for the lifespan of local solutions to the corresponding Cauchy problem. The technique in the proofs consists in studying the growth (with respect to the time variable) of the spatial average of a local solution. A special emphasis is given to a threshold case for the parameters appearing in the nonlinear term: by combining an integral representation formula by Yagdjian-Galstian [4] with a technique borrowed from the critical case for the classical wave equation [3], we prove that local solutions blow-up even in this threshold case.

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Stability properties of ultraholomorphic classes of Roumieu-type defined by weight matrices

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We characterize several stability properties, such as inverse or composition closedness, for ultraholomorphic function classes of Roumieu-type defined in terms of a weight matrix. In this way we transfer and extend known results from J. Siddiqi and M. Ider, from the weight sequence setting and in sectors not wider than a halfplane, to the weight matrix framework and for sectors in the Riemann surface of the logarithm with arbitrary opening. The key argument rests on the construction, under suitable hypotheses, of characteristic functions in these classes for unrestricted sectors. As a by-product, we obtain new stability results when the growth control in these classes is expressed in terms of a weight sequence, or of a weight function in the sense of Braun-Meise-Taylor.

Propagation of anisotropic Gabor singularities for Schrödinger type equations

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We show results on propagation of anisotropic Gabor wave front sets for solutions to a class of evolution equations of Schrödinger type. The Hamiltonian is assumed to have a real-valued principal symbol with the anisotropic homogeneity $a(\lambda x, \lambda^\sigma \xi) = \lambda^{1+\sigma} a(x, \xi)$ for $\lambda > 0$ where $\sigma > 0$ is a rational anisotropy parameter. We prove that the propagator is continuous on anisotropic Shubin–Sobolev spaces. The main result says that the propagation of the anisotropic Gabor wave front set follows the Hamilton flow of the principal symbol.