ALCUNE RIFLESSIONI SULLE CONICHE

Emilia Florio

UNIVERSITÀ DELLA CALABRIA
DIPARTIMENTO DI MATEMATICA E INFORMATICA

APOLLONII

PERGAEI CONICORVM
LIBRI QVATTVOR.

VNA CVM PAPPI ALEXANDRINI
LEMMATIBVS, ET COMMENTARIIS
EVTOCII ASCALONITAE.

SERENI ANTINSENSIS
PHILOSOPHI LIBRI DVO
NVNC PRIMVM IN LVCEM EDITI.

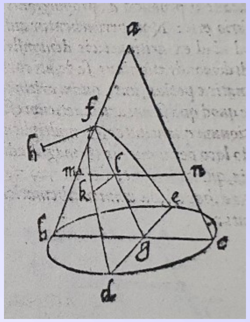
QVAE OMNIA NVPER FEDERICVS
Commandinus Vrbinas mendis quamplurimis expurgata è Græco conuertit, & commentariis illuſtrauit.

CVM PRIVILEGIO PII IIII. PONT. MAX.
IN ANNOS X.

BONONIAE,
EX OFFICINA ALEXANDRI BENATII.
M D LXVI.

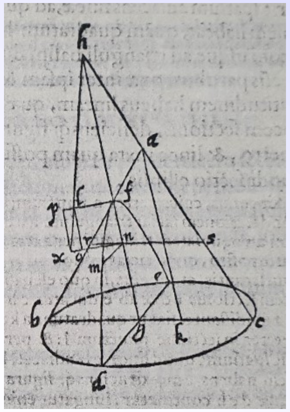
THEOREMA XI. PROPOSITIO XI.

Si conus plano per axem ſecetur: ſecetur autem & altero plano ſecante baſim coni ſecundum rectam lineam, quæ ad baſim trianguli per axem ſit perpendicularis: & ſit diameter ſectionis uni laterum trianguli per axem æquidiſtans: recta linea, quæ à ſectione coni ducitur æquidiſtans communi ſectioni plani ſecantis, & baſis coni, uſque ad ſectionis diametrum; poterit ſpatium æquale contento linea, quæ ex diametro abſciſſa inter ipſam & uerticē ſectionis interiicitur, & alia quadam, quæ ad lineam inter coni angulum, & uerticem ſectionis interiectam, eam proportionem habeat, quàm quadratum baſis trianguli per axem, ad id quod reliquis duobus trianguli lateribus continetur. dicatur autem huiuſmodi ſectio parabole.



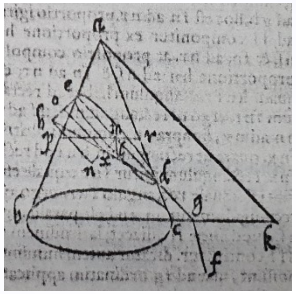
THEOREMA XII. PROPOSITIO XII.

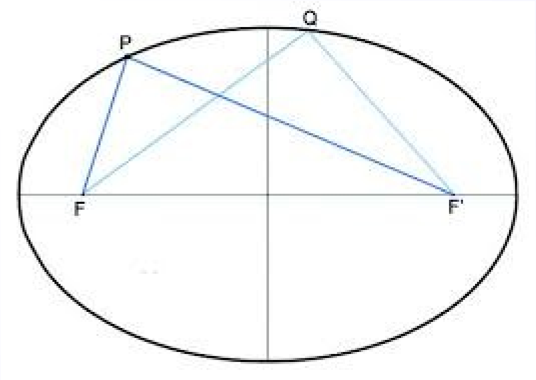
Si conus plano per axem ſecetur; ſecetur autem & altero plano ſecante baſim coni ſecundum rectam lineam, quæ ad baſim trianguli per axem ſit perpendicularis: & ſectionis diameter producta cum uno latere trianguli per axem, extra uerticem coni conueniat: recta linea, quæ à ſectione ducitur æquidiſtans communi ſectioni plani ſecantis, & baſis coni uſque ad ſectionis diametrum, poterit ſpatium adiacens lineæ, ad quam ea, quæ in directum conſtituitur diametro ſectionis, ſubtenditurq; angulo extra triangulum, eandem proportionem habet, quam quadratum lineæ, quæ diametro æquidiſtans à uertice ſectionis uſque ad baſim trianguli ducitur, ad rectangulum baſis partibus, quæ ab ea fiunt, contentum: latitudinem habens lineam, quæ ex diametro abſcinditur, inter ipſam & uerticem ſectionis interiectam; excedensq; figura ſimili, & ſimiliter poſita ei, quæ continetur linea angulo extra triangulum ſubtenſa, & ea, iuxta quam poſſunt quæ ad diametrum applicantur. uocetur autem huiuſmodi ſectio hyperbole.

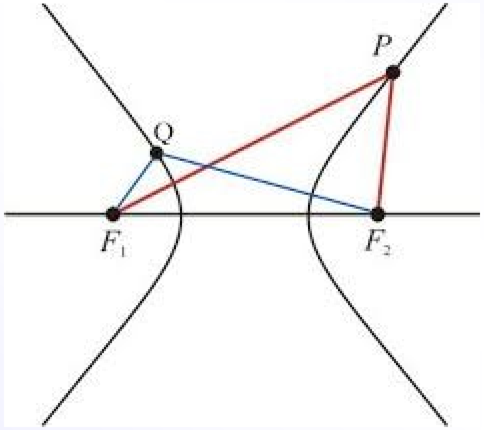


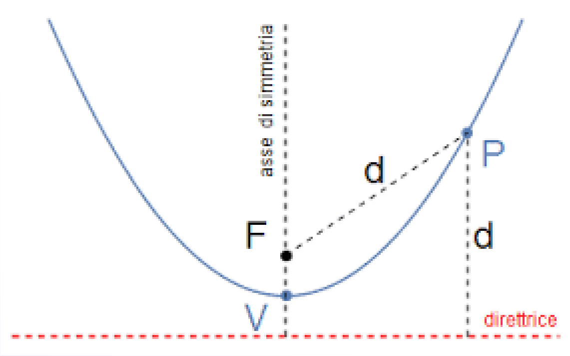
THEOREMA XIII. PROPOSITIO XIII.

Si conus plano per axem ſecetur, & ſecetur altero plano conuenienté cum utroque latere trianguli per axem, quod neque baſi coni æquidiſtet, neque ſubcontrarie ponatur: planum autem, in quo eſt baſis coni, & ſecans planum conueniant ſecundum rectam lineam, quæ ſit perpendicularis uel ad baſim trianguli per axem, uel ad eam, quæ in directum ipſi conſtituitur: recta linea, quæ à ſectione coni ducitur æquidiſtans communi ſectioni planorum uſque ad diametrum ſectionis poterit ſpatium adiacens lineæ, ad quam ſectionis diameter eam proportionem habeat, quam quadratum lineæ diametro æquidiſtantis à uertice coni uſque ad trianguli baſim ductæ, habet ad rectangulum contentum baſis partibus, quæ inter ipſam & rectas trianguli lineas interiiciuntur; latitudinem habens lineam, quæ ex diametro ab ipſa abſcinditur ad uerticem ſectionis, deficiensq; figura ſimili, & ſimiliter poſita ei, quæ diametro, & linea iuxta quam poſſunt, continetur. dicatur autem huiuſmodi ſectio ellipſis.




P
Q
F
F'


P
Q
F_1
F_2


asse di simmetria
d
P
F
d
V
direttrice

Come colleghiamo la presentazione delle coniche

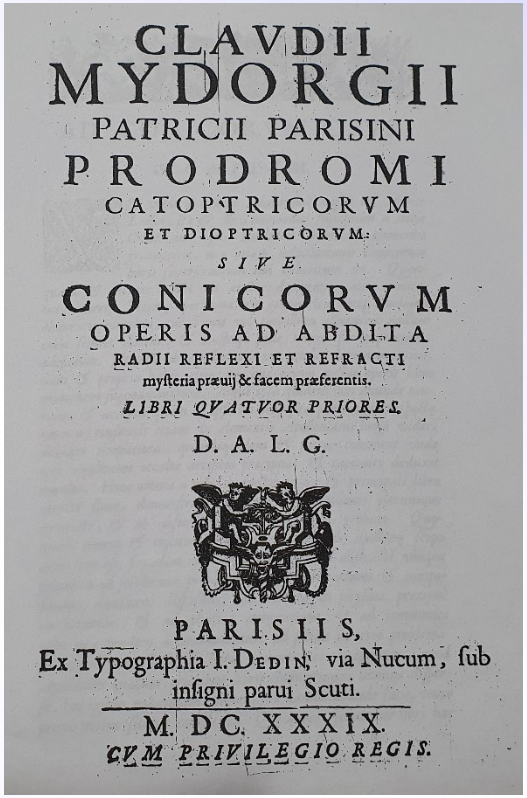
come luoghi di punti di un piano

con quanto proviene dalla tradizione?

Dalle coniche come sezioni di un cono

alle coniche come curve in un piano

Claude Mydorge (1585-1647)



CLAVDII
MYDORGII
PATRICII PARISINI
PRODROMI
CATOPTRICORVM
ET DIOPTRICORVM:
SIVE
CONICORVM
OPERIS AD ABDITA
RADII REFLEXI ET REFRACTI
mysteria præuij & facem præferentis.
LIBRI QVATVOR PRIORES.
D. A. L. G.

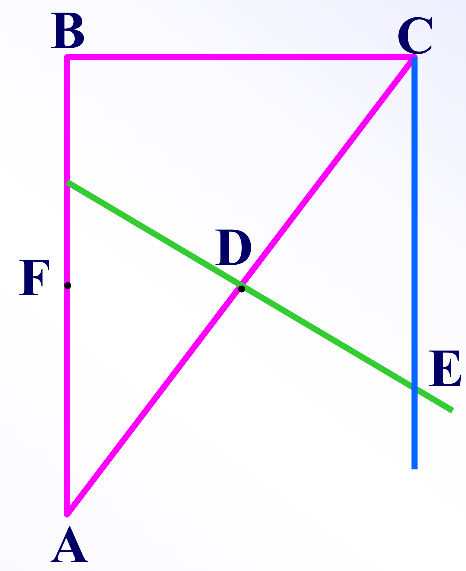
PARISIIS,
Ex Typographia I. DEDIN, via Nucum, sub
insigni parui Scuti.
M. DC. XXXIX.
CVM PRIVILEGIO REGIS.

occurret. Norma operi vna erit geometrica ratio Euclideis elementis contenta, cui ſi quis vel tantillùm aſſueuerit nullum inibi ſentiet obicem, & inoffenſo pede totum facile percurret ædificium.

Hinc omnia à nobis in eodem primo & principali libro directè ſunt demonſtrata, quæ ab ipſo Apollonio plerumque indirectè, & ab abſurdi conſequentia ſunt probata. Quoniam autem & inſtitutum noſtrum longè ab Apollonij ſcopo diuerſum eſt, ſiquidem ille Conica ſua purè & abſtractè vbique propoſita ad ſolidorum problematum determinationes & compoſitiones duntaxat diſpoſuit: nos Conica in phyſicis præcipuè conſectamur, & ad phyſica componimus, vt ad communes vſus ipſa tandem accommodemus: ad ſolida interim problemata etiamnum ſatis factura procurantes: eapropter, quod Apollonius quatuor prioribus libris elementorum nomine diſperſit, hoc ipſum maximam partem, ſed tamen abunde ſatis, vnico primo noſtro ſumus complexi.

THEOR. VIII. PROP. XVII.

Si ſit triangulum quodcunque ABC rectangulum ad B: ſectæque AC bifariam in D perpendicularis excitetur DE, cui à puncto C ducta CE ipſi AB parallela occurrat in E: ſeceturque AB bifariam in F: Dico punctum E eſſe in eadem parabola cuius vertex ſit F, & vmbilicus A.



$A\hat{B}C = 90°$ $AD = DC$ $DE \perp AC$ $CE \parallel AB$ $AF = FB$

Sia G il punto di intersezione di DE con AB.

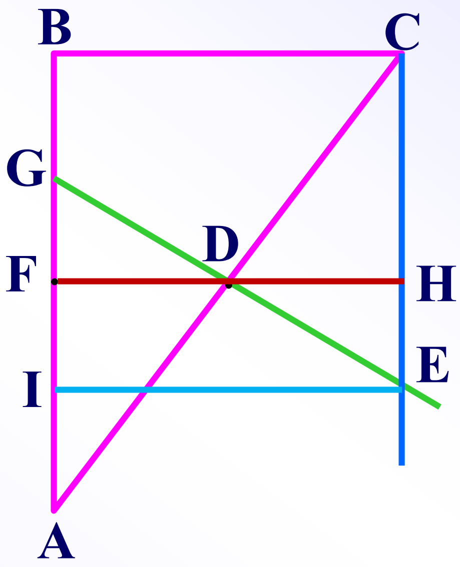
Si tracci FH parallela a BC.

I triangoli FDA e HDC sono congruenti, quindi FD = DH.

I triangoli FGD e HDE sono congruenti, quindi HE = FG.

Si tracci EI parallela a BC.

Allora: HE = IF = FG.



Nel triangolo ADG, rettangolo in D, si ha:

$$\mathbf{FD^2 = AF \cdot FG}$$

$$\mathbf{FD^2 = AF \cdot IF}$$

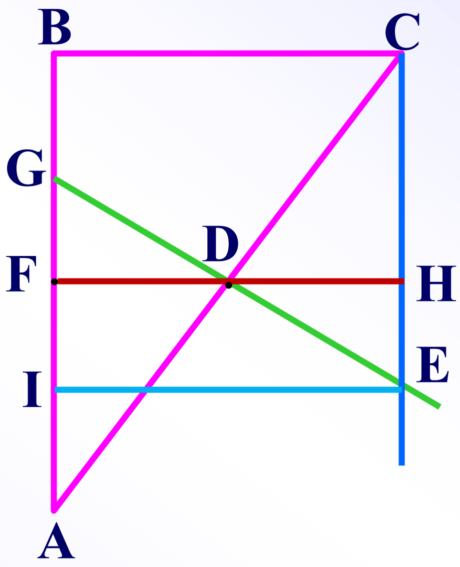
$$\mathbf{4 \cdot FD^2 = 4 \cdot AF \cdot IF}$$

$$\mathbf{(2FD)^2 = IF \cdot 4AF}$$

Essendo 2FD = FH = EI, si ottiene:

$$\mathbf{EI^2 = IF \cdot 4AF}$$

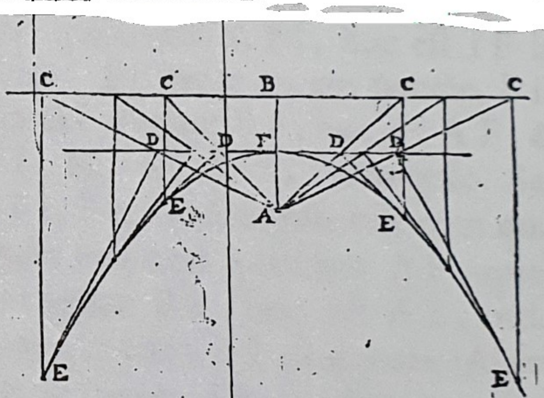
Dunque il punto E si trova sulla parabola di vertice F e fuoco A.



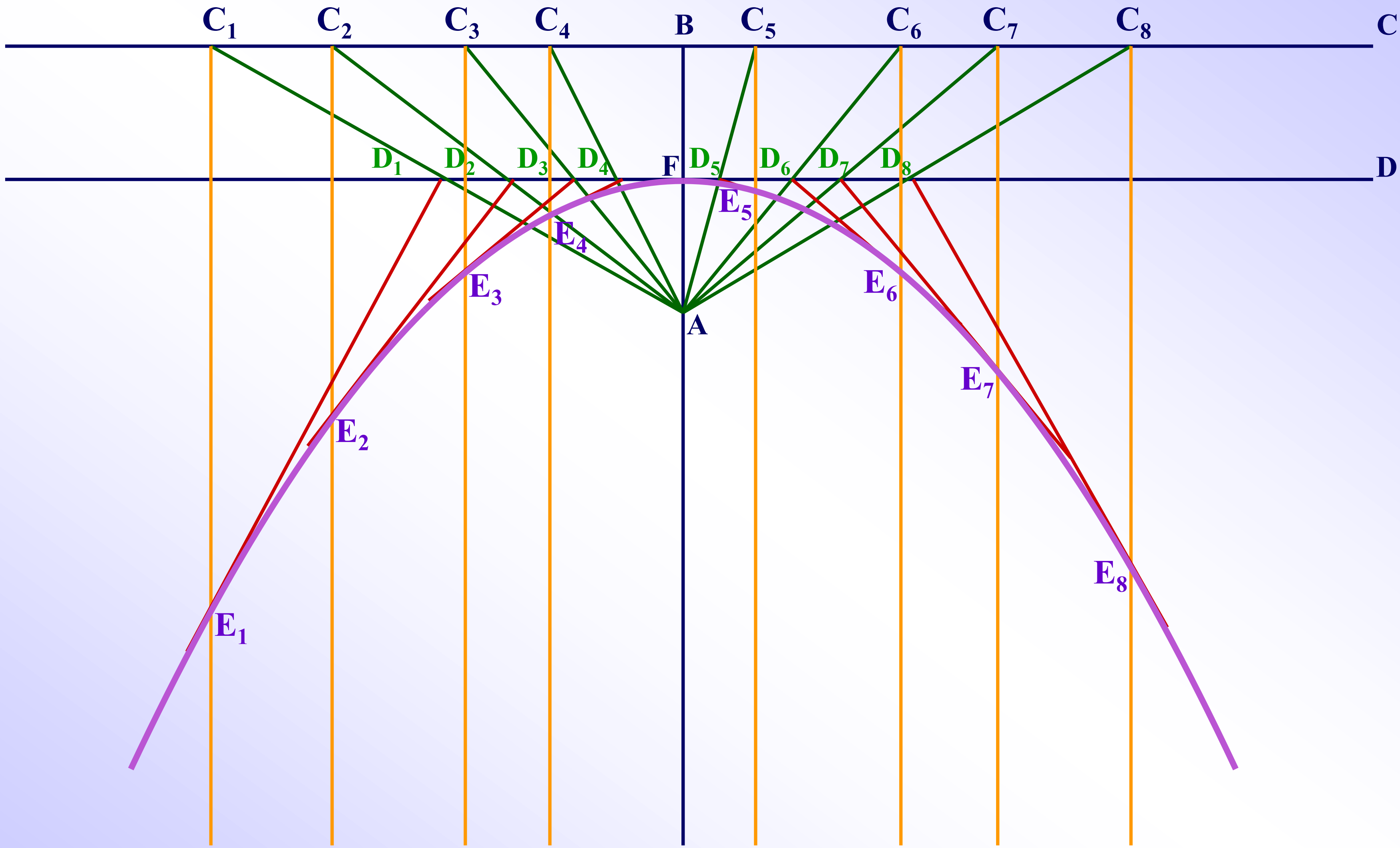
PROP. XIX.

Datis, poſitione, paraboles vmbilico, & vertice; parabolem in eodem plano per puncta deſcribere.

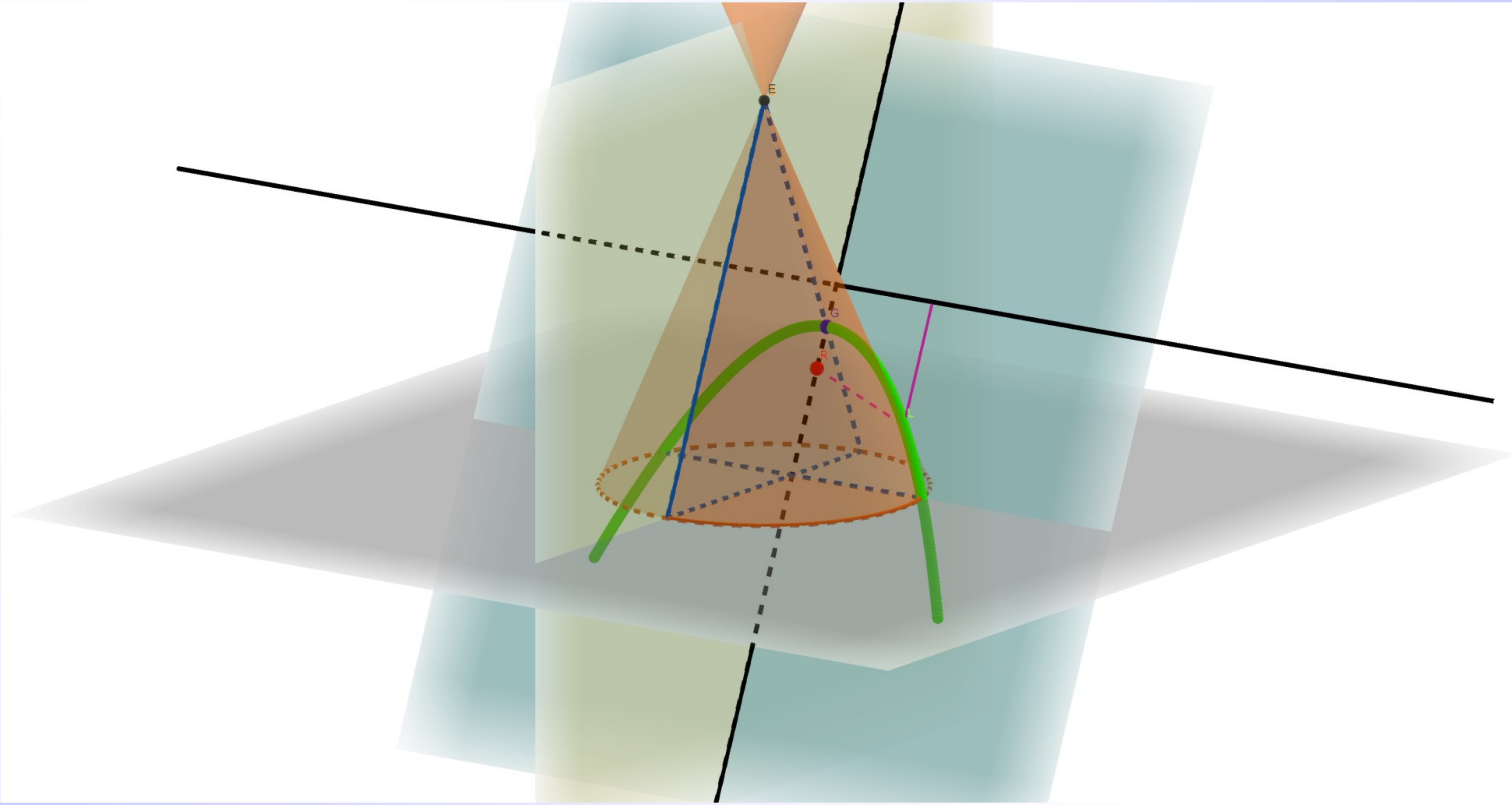
Sit datus A paraboles vmbilicus, eiuſque vertex F.



Iuncta AF producatur in B, vt ſit FB æqualis AF: & à punctis F & B ipſi AB perpendiculares excitentur FD, BC productæ quantum opus erit: ductis autem à puncto A quotcunque rectis, vt AC, ſecantibus rectam FD, vt in D, perpendiculares totidem ijſdem erigantur, vt DE, occurrentes ductis à punctis C ipſi AB parallelis, vt CE, in E. Palam igitur eſt, ex 17 huius, inuenta eiuſmodi puncta, vt E, in eadem eſſe parabola cuius ſit vertex F, & vmbilicus in A.



C1
C2
C3
C4
B
C5
C6
C7
C8
C
D1
D2
D3
D4
F
D5
D6
D7
D8
D
E1
E2
E3
E4
E5
E6
E7
E8
A


E
G
R

Dalle coniche come curve in un piano

alle coniche come sezioni di un cono

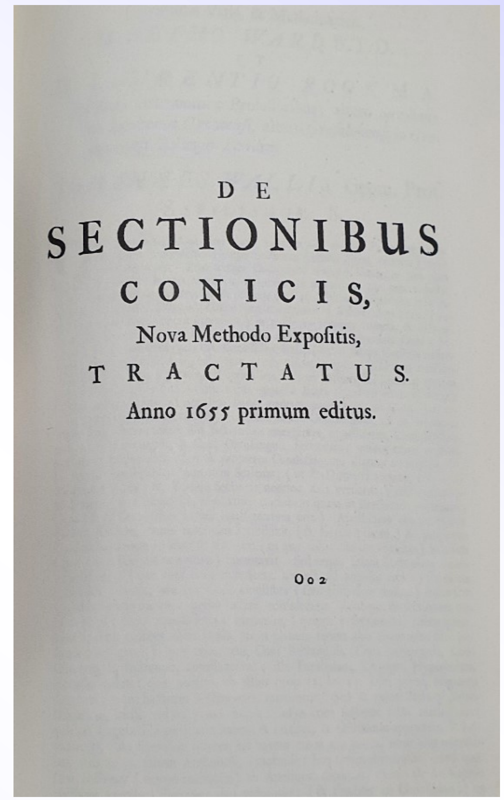
Johannis Wallis S. T. D.
Geometriæ Professoris *SAVILIANI*,
in Celeberrima Academia OXONIENSI;

OPERA MATHEMATICA.

Volumen Primum.

OXONIÆ,
E THEATRO SHELDONIANO MDCXCV.

John Wallis (1616-1703)



DE
SECTIONIBUS
CONICIS,
Nova Methodo Expositis,
TRACTATUS.
Anno 1655 primum editus.

Oo 2

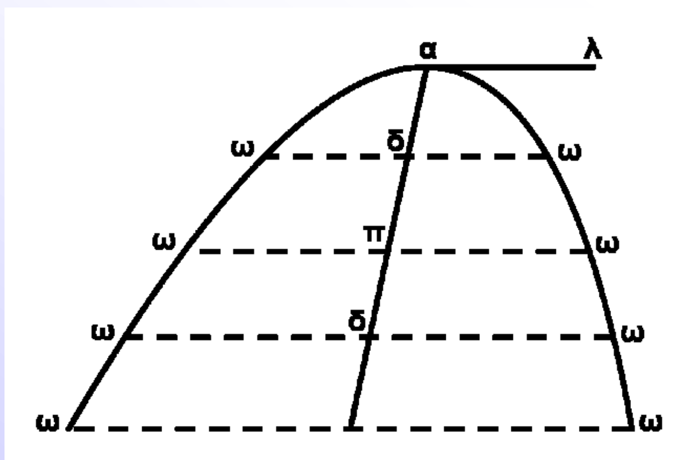
Sia data la parabola piana αω

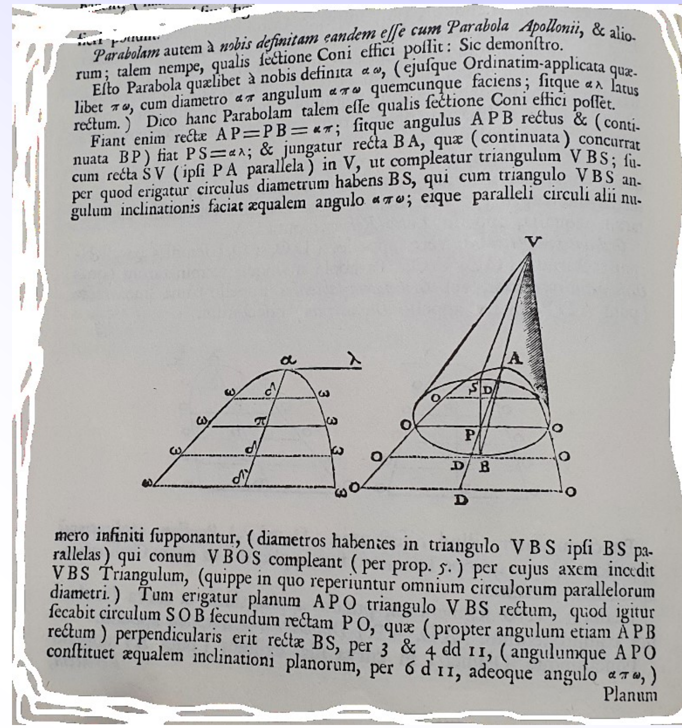
le cui ordinate siano πω

il diametro απ

l'angolo tra l'ordinata e il diametro απω

il lato retto αλ





Parabolam autem à *nobis definitam eandem eſſe cum Parabola Apollonii*, & aliorum; talem nempe, qualis ſectione Coni effici poſſit: Sic demonſtro.

Eſto Parabola quælibet à nobis definita αω, (ejuſque Ordinatim-applicata quælibet πω, cum diametro απ angulum απω quemcunque faciens; ſitque αλ latus rectum.) Dico hanc Parabolam talem eſſe qualis ſectione Coni effici poſſet.

Fiant enim rectæ AP=PB=απ; ſitque angulus APB rectus & (continuata BP) fiat PS=αλ; & jungatur recta BA, quæ (continuata) concurrat cum recta SV (ipſi PA parallela) in V, ut compleatur triangulum VBS; ſuper quod erigatur circulus diametrum habens BS, qui cum triangulo VBS angulum inclinationis faciat æqualem angulo απω; eique paralleli circuli alii numero infiniti ſupponantur, (diametros habentes in triangulo VBS ipſi BS parallelas) qui conum VBOS compleant (per prop. 5.) per cujus axem incedit VBS Triangulum, (quippe in quo reperiuntur omnium circulorum parallelorum diametri.) Tum erigatur planum APO triangulo VBS rectum, quod igitur ſecabit circulum SOB ſecundum rectam PO, quæ (propter angulum etiam APB rectum) perpendicularis erit rectæ BS, per 3 & 4 dd 11, (angulumque APO conſtituet æqualem inclinationi planorum, per 6 d 11, adeoque angulo απω,)

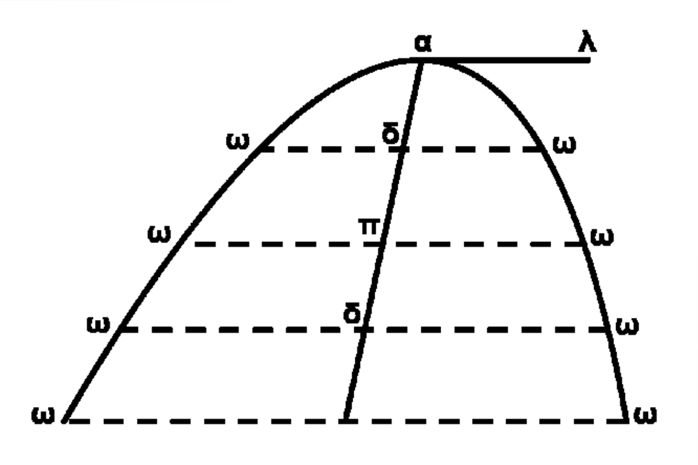
Planum

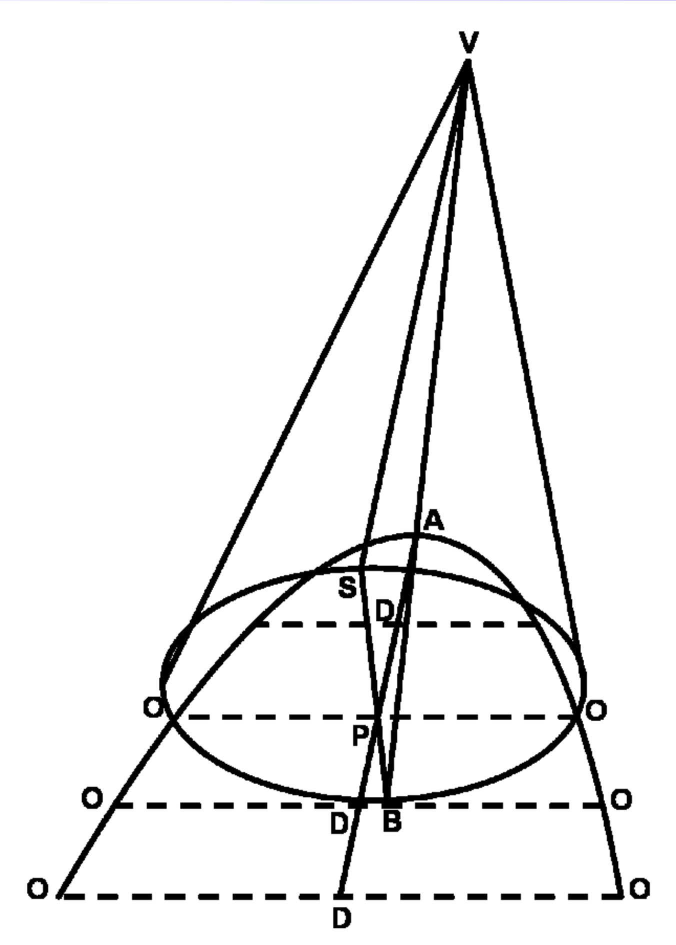
Sia

$$AP = PB = \alpha\pi$$

$$A\widehat{P}B = 1R$$

$$PS = \alpha\lambda$$

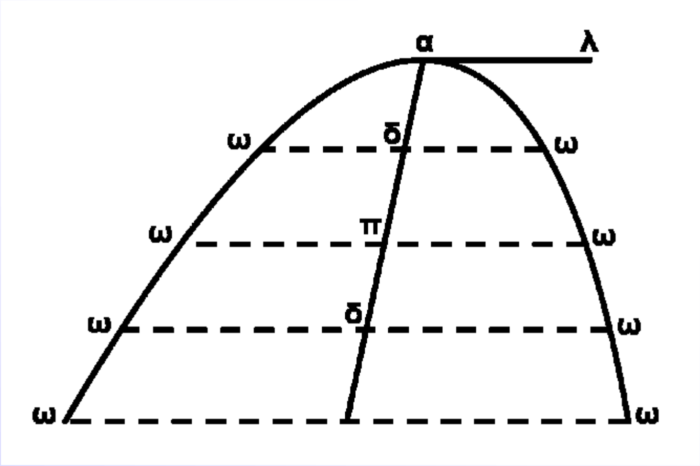


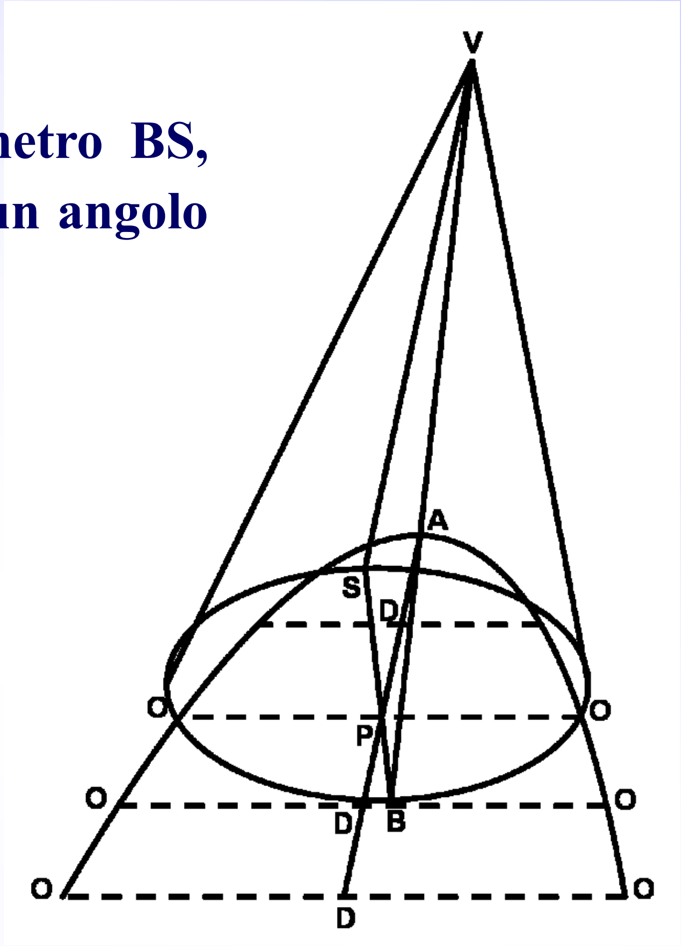


Dal punto S si tracci la parallela a PA.

Tale parallela intersechi la retta passante per i punti B e A nel punto V.

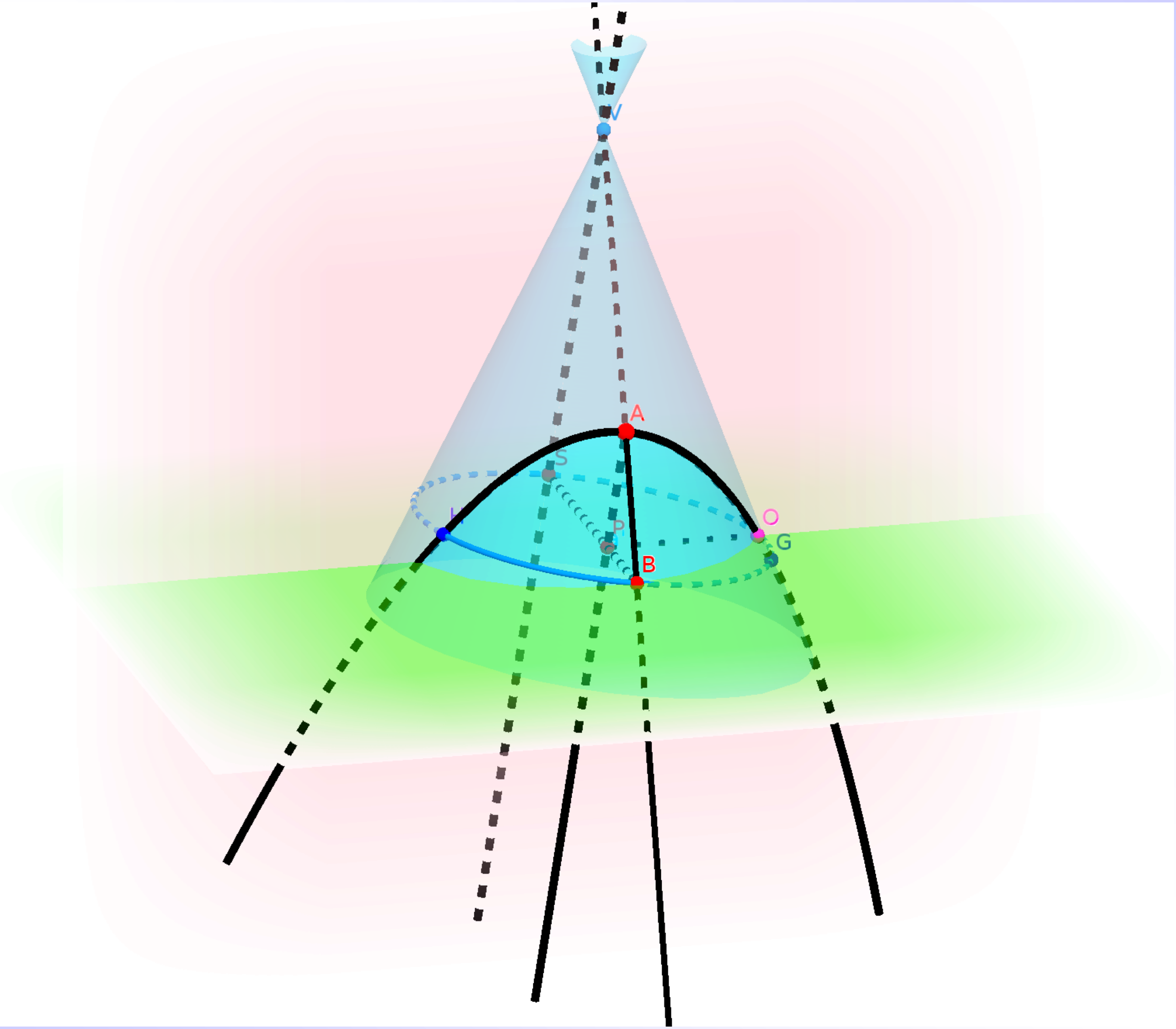
Si costruisca il cerchio di diametro BS, che formi con il triangolo VBS un angolo uguale a $\alpha\pi\omega$





Si può supporre l'esistenza di infiniti cerchi paralleli, aventi il diametro parallelo a BS.

Tali cerchi individuano il cono VBOS per il cui asse passa il triangolo VBS.


V
A
S
H
P
O
G
B

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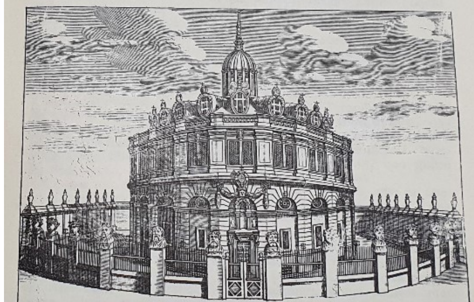

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